Roll No. ....   **Total Pages : 06**

MAR-21-210074

B. Tech. EXAMINATION, March 2021

Semester I (NS)

ENGINEERING MATHEMATICS—I

(Common for Gp A & B)

NS-101

Time : 3 Hours   *Maximum Marks :* 100

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.

Note : Attempt *Five* questions in all, selecting *one* question from each Sections A, B, C and D. Q. No. **9** is compulsory.

Section A

1. (a) Reduce the matrix $A = \begin{bmatrix} 1 & 2 & 1 & 0 \\ -2 & 4 & 3 & 0 \\ 1 & 0 & 2 & -8 \end{bmatrix}$ into

normal form and hence find the rank of the matrix A. 5

(b) Find the values of λ and μ for which the simultaneous equations :

$$x + y + z = 6$$

$$x + 2y + 3z = 10$$

$$\text{and} \quad x + 2y + \lambda z = \mu$$

have :

(i) no solution

(ii) a unique solution.

(iii) an infinite number of solutions. 5

2. (a) Verify the Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and hence find A^{-1}.

5

(b) Prove that the diagonal elements of a skew-Hermitian matrix are either purely imaginary or zero. 5

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Section B

3. (a) If $x^x y^y z^z = k$ (constant), show that : **5**

$$x = y = z,\ \frac{\partial^2 z}{\partial x \partial y} = -(x \log ex)^{-1}.$$

(b) If $u = \log\left(\frac{x^2 + y^2}{\sqrt{x} + \sqrt{y}}\right)$, find the value of :

(i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$

(ii) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$. **5**

4. (a) If $u = \frac{x+y}{1-xy}$ and $v = \tan^{-1}(x) + \tan^{-1}(h)$, find $\frac{\partial(u,v)}{\partial(x,y)}$. Are u and v functionally related ? If so find the relationship. **5**

(b) Find the maximum and minimum value of the function $f(x, y) = 3x^2 + y^2 - x$ over the region $2x^2 + y^2 \leq 1$. **5**

Section C

5. (a) Evaluate : **5**

$$\int_0^a x^2\left(a^2 - x^2\right)^{3/2} dx.$$

(b) Find the length of the curve $y = \log \sec x$ between the points $x = 0$ and $x = \frac{\pi}{3}$. **5**

6. (a) Transform the internal :

$$\int_0^a \int_0^{\sqrt{a^2 - x^2}} y^2 \sqrt{x^2 + y^2}\ dx\,dy$$

by changing to polar coordinates and hence evaluate it. **5**

(b) Evaluate the triple integral of the function $f(x, y, z) = x^2$ over the region V enclosed by the places $x = 0$, $y = 0$, $z = 0$ and $x + y + z = a$. **5**

Section D

7. (a) Find the value of $\sqrt{-5 + 12i}$, where $i = \sqrt{-1}$. **5**

(b) Prove that : 5

$$\left(\frac{1+\sin\alpha+i\cos\alpha}{1+\sin\alpha-i\cos\alpha}\right)^n=\cos\left(\frac{n\pi}{2}-n\alpha\right)+i\sin\left(\frac{n\pi}{2}-n\alpha\right).$$

8. (a) If α, α^2, α^3, α^4 are roots of $x^5 - 1 = 0$, then show that : 5

$$(1-\alpha)(1-\alpha^2)(1-\alpha^3)(1-\alpha^4)=5.$$

(b) Find the sum of the series : 5

$$x\sin\theta+\frac{x^2}{2!}\sin 2\theta+\frac{x^3}{3!}\sin 3\theta+\ldots\ldots \text{ upto } \infty.$$

(Compulsory Question)

9. (a) Reduce the matrix $A=\begin{bmatrix}1&2&1&2\\1&3&2&2\\2&4&3&4\\3&7&4&6\end{bmatrix}$ to echelon form and hence find its rank. 2

(b) Obtain the matrix of the quadratic form : 2

$$Q=x_1^2+2x_2^2-7x_3^2-4x_1x_2+8x_1x_3+5x_2x_3.$$

(c) Find the eigen values and eigen vectors of the matrix $A=\begin{bmatrix}5&4\\1&2\end{bmatrix}$. 2

(d) If $z=e^{ax+by}.f(ax-by)$, show that : 2

$$b\frac{\partial z}{\partial x}+a\frac{\partial z}{\partial y}=2abz.$$

(e) Find the Jacobian $\frac{\partial(u,v)}{\partial(x,y)}$, where $u=e^x\sin y$, $v=x+\log\sin y$. 2

(f) Evaluate : 2

$$\int \sec^4 x\,dx.$$

(g) Find the area of the region bounded by the curves $xy = 2$, $4y = x^2$, $y = 4$. 2

(h) Prove that : 2

$$\frac{\beta(m+1,n)}{\beta(m,n)}=\frac{m}{m+n}.$$

(i) Find the value of $\log_{-3}(-2)$. 2

(j) Separate real and imaginary parts of $(1+i)^i$. 2